EXPANSION OF MOVING UNIFORM FLUIDIZED BEDS OF POLYDISPERSE GRANULATED MATERIALS

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Critical review is presented of the literature on expansion of moving uniform fluidized beds of granulated materials. The published relations are critically examined using experimental data on fluidization of various mixtures of narrow size fractions of sand by water. A procedure is proposed enabling a reliable estimate of the composition of uniform moving fluidized beds for both the counter- and co-current motion of the particles and the fluid. The procedure allows also that one of the components of the granulated material may not be moving. The derived relations pertain to granulated materials of different size, density and of the shape particles in the mixture.

This work is a continuation of the paper on expansion of moving fluidized beds of monodisperse materials[1]. To verify the expressions describing expansion of moving polydisperse fluidized beds binary mixtures of narrow size fractions of sand were fluidized by water. The experimental set-up has been described in one of the preceding communication[1]. The description is formulated in terms of scalar quantities since we have confined ourselves to unidimensional motion. These scalar quantities take on positive values for the direction opposite to the direction of gravitational acceleration.

As a co-current arrangement of the fluidized bed we take such an arrangement in which the direction of the over-all velocity of an arbitrary component of the solid particles is the same as the direction of mean fluid velocity. Analogously, a counter-current aragement of a moving fluidized bed is such in which the over-all velocity of an arbitrary component of the particle mixture is opposite to the direction of mean fluid velocity. In the experimental part of the work we have restricted ourselves to co-current arrangement with the positive direction of motion of both the particles and the fluid, and the counter-current arrangement with the negative direction of motion of the particles and positive direction of the fluid flow.

An attempt to describe expansion of moving fluidized beds of polydisperse materials has been made by Gasparjan and Zaminjan[2], and Finkelstein, Letan and Elgin[3]. For expansion of co-currently moving fluidized beds of polydisperse materials the authors[2] assume that: *a*) the density of solid particles is higher than that of the fluid, *b*) constant column cross section, *c*) both phases are incompressible, *d*) the process is steady, and *e*) the polydisperse material is a mixture of monodisperse components (the particle diameter distribution function has a form of "staircase").

In the derivation of their expressions the authors started from the situation when a binary mixture of solid particles and liquid moves upwards through the column. The particles lag behind the liquid and finer particles proceed faster than the coarse ones. On the basis of cit.[4] the authors[2] further adopt the hypothesis according to which the superficial velocity of liquid with respect to individual components of the bidisperse material is given by the following form of the expansion function

$$f(\mathrm{Re}, \mathrm{Ar}, \varepsilon) = 0\,. \tag{1}$$

The porosity for individual components is taken equal that of the polydisperse material.

The authors[2] extended the results for bidisperse systems to polydisperse materials in co-current arrangement of the moving fluidized bed in the form of a single equation. As a drawback of this expression appears its restriction to co-current arrangement and tedious solution.

Using the same starting assumptions[2] but a different approach we have derived following equation describing expansion of an arbitrary component of a uniform moving fluidized bed of polydisperse material

$$w_i = w_f + u_c[1 - Y_i/[X_i(1 - \varepsilon)]]\,, \quad i = 1, 2, \ldots, k\,;\; u_c = \dot{V}_s/S\,, \tag{2}$$

where

$$\sum_{i=1}^{k} X_i = 1\,, \quad Y_i = \dot{V}_{si}/\dot{V}_s\,, \quad \sum_{i=1}^{k} Y_i = 1\,. \tag{3}$$

Equation (*2*) written for all components of the polydisperse material together with the expansion function, Eq. (*1*), of these components plus Eq. (*3*) serve to describe expansion of moving fluidized bed of polydisperse material of known composition.

The single equation of the authors[2] is thus replaced with advantage by a set of equations following from Eq. (*2*). The described approach suits both for the co- and counter-current arrangement of the flow. The generalization was achieved by replacing the vectors by oriented scalar quantities. Moreover, Eq. (*2*) may be used even for cases when one of the components of the polydisperse mixture in the fluidized bed does not move. In such case one thus has for the n-th non-moving component that $Y_n = 0$ ($i = n$), since this component cannot be contained in the feed and must have got into the bed in the period prior to the observation. For clarity let us note that $X_n \neq 0$ as the n-th component is a part of the solid phase of the fluidized bed.

Let us still examine the form of Eq. (*2*) for $k = 1$, *i.e.* for the case of monodisperse moving fluidized bed.

After some arrangement one obtains (omitting the subscript 1)

$$w = w_f - u_c[\varepsilon/(1 - \varepsilon)]\,. \tag{2a}$$

Eq. (*2a*) holds only for ideal uniformly moving fluidized bed, *i.e.* a bed of constant porosity along the column height and over the column cross section. The same condition must be met also in case of Eq. (*2*).

This theory[2] has been experimentally verified[5] on a co-currently moving fluidized bed in which both the particles and the fluid moved in the positive direction. The experiments were carried out with various bi- and tridisperse materials fluidized by water under different experimental arrangement. The authors[5] have concluded that their approach to the description of the moving fluidized bed provides good agreement with the experiments. However, no more profound analysis can be afforded because the papers[2,5] do not indicate individual experimental points.

Expressions describing moving fluidized bed of polydisperse materials different from those mentioned above[2] and from ours have been derived by Finkelstein, Letan and Elgin[3]. Using the results taken over from other communications[6,7] the authors[3] adopted the assumption that in short moving fluidized beds no particle segregation exists after these had entered the bed provided that the average residence time does not exceed 60 seconds. From this assumption it follows

$$u_{ci}/(1 - \varepsilon_i) = \text{const.}\,, \quad i = 1, 2, \ldots, k\,. \tag{4}$$

The composition of a polydisperse particle mixture in the moving fluidized bed is expressed by a single quantity, namely the average diameter d_a

$$d_a = \sum_{i=1}^{k} (1 - \varepsilon_i) / \sum_{i=1}^{k} (1 - \varepsilon_i)/d_i\,. \tag{5}$$

Simultaneously it holds that

$$\varepsilon = 1 - \sum_{i=1}^{k} (1 - \varepsilon_i)\,. \tag{6}$$

The expansion of the moving fluidized bed is described by the authors[3] as an expansion of monodisperse particles of diameter d_a. For this process the authors[3] use Eq. (*2a*). This approach was tested experimentally by the authors[3] by fluidizing various binary mixtures of granular materials by water. The arrangement of the experiments was both co- and counter-current; the direction of motion of the particles was always negative. The experiments were designed to measure local porosity along the column starting from the distance about 300 mm from the feed. In the examined section the authors[3] found the porosity to be constant. This lead them to an incorrect conclusion that no particle segregation exists in the bed after these had entered the fluidized bed. According to Kwauk[8] it is very likely that partial segregation did occur already in the entrance section which was not examined. Our observation also indicates that a significant segregation occurs within already a few seconds. Thus the assumption of the authors[3] taking 60 seconds as a limit below which no segregation occurs is irrealistic. Moreover, the deviations of the experimental and calculated values[3] amount to 7 ÷ 17%, a fact accounted for by the authors themselves by a certain degree of segregation.

The difference between the composition of the solids within the fluidized bed and the feed under various conditions can be further illustrated by Eq. (*2*). For simplicity we shall consider only binary particle mixtures. On writting Eq. (*2*) for the two components one has two equations the combination of which gives

$$Y_J = X_J + X_J(1 - X_J)(w_H - w_J)(1 - \varepsilon)/u_c\,. \tag{2b}$$

The dependence given by Eq. (*2a*) is shown graphically in Fig. 1 for a binary mixture of fractions F3 and F6 of sand (physical properties of these fractions have been published in the preceding paper[1]), constant porosity equal $\varepsilon = 0{\cdot}8$ and $w_f > 0$.

The straight line in Fig. 1 evidences the agreement between the composition of the fed mixture, Y_J, and the composition of the solids in the fluidized bed, X_J. This relation is consistent with the concept of the authors[3]. From Fig. 1 it is apparent that the mentioned concept[3] is real, approximately according to Eq. (2), only for high absolute values of throughput, u_c, of the solid material through the column regardless of the residence time of the particles in the fluidized bed. On the contrary, substantial segregation after entering the column may be expected, according to Eq. (2), at low absolute throughputs u_c. Similar conclusions could be obtained by analysis of Eq. (2b) for constant value of u_c and variable value of $(1 - \varepsilon)$ or $(w_H - w_J)$.

The authors[2,3,5] do not give enough information in their papers about the experimental conditions and the results. Accordingly, Eq. (2) and the approach used by the authors[2,3] was verified on the basis of our own experimental data.

EXPERIMENTAL

The essential part of the experimental set-up used was a 50 mm in diameter cylindrical column equipped with a grid[1]. The validity of the relations describing expansion of the moving uniform fluidized bed of polydisperse material was tested on two binary mixtures of narrow fractions of sand (F2 and F4) and (F3 and F6). The characteristics of the fractions used have been given earlier[1]. The experiments were carried out with an moving fluidized bed under co- and counter-current arrangement of the flows always with a positive direction of the flow of water. A part of the experiments was carried out with a non-moving coarse fraction passed through co-currently by water and the fine fraction. The expansion of the moving fluidized bed was detected from pressure differences along the fluidized bed. In addition, the size distribution of the fed material as well as the particles of the fluidized bed were measured. A total of 28 experiments were performed of which 10 were under co-current and 10 under counter-current arrangement, 8 with one non-moving component. The absolute value of the throughput was in all experiments $u_c \in \langle 0{\cdot}25; 2{\cdot}03 \rangle$ mm/s.

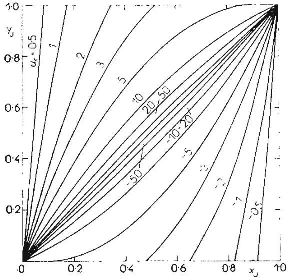

FIG. 1

A plot of Composition of Fed Binary Mixture Y_J *versus* Solid Phase Composition in the Moving Fluidized Bed X_J

$\varepsilon = 0{\cdot}8$, $w_f > 0$, $\varrho_s > \varrho_f$, $Ar_J = 2549$, $Ar_H = 17645$.

The principal conditions of the experiments and the results are summarized in Table I. Individual experiments are marked by either P, D or PN indicating respectively the co-current, the counter-current and the experiments with non-moving component. The composition of the binary mixture is indicated by two digits following a letter character; the sequence of the experiment of a given type is shown by a figure preceding the letter. For instance, an experiment 6P36 is the sixth experiment under co-current arrangement of the flow with the binary mixture F3 + F6.

TABLE I

Experimental Data and their Comparison with the Results of Theoretical Relations

Experiment	u_c mm/s	w_f mm/s	Y_J	$\varepsilon_{M(E)}$	$\varepsilon_{M(C)}$		Δ_M	
					cit.[3]	Eq. (2)	cit.[3]	Eq. (2)
1P36	0·39	27·13	0·519	0·586	0·646	0·585	+10·2	−0·1
2P36	0·67	34·71	0·464	0·625	0·699	0·635	+11·8	+1·6
3P36	0·37	27·15	0·647	0·591	0·684	0·586	+15·7	−0·8
4P36	0·71	34·67	0·497	0·630	—	0·635	—	+0·8
5P36	1·25	43·45	0·490	0·676	—	0·687	—	+1·6
6P36	1·65	55·98	0·500	0·742	0·818	0·752	+10·2	+1·3
1D36	−1·39	17·71	0·538	0·679	—	0·662	—	−2·5
2D36	−1·55	29·07	0·529	0·801	0·722	0·777	− 9·9	−3·0
3D36	−1·53	36·91	0·487	0·820	0·777	0·847	− 5·2	+3·3
5D36	−1·87	14·72	0·516	0·644	—	0·629	—	−2·3
6D36	−1·81	18·04	0·518	0·683	—	0·670	—	−1·9
7D35	−2·03	23·29	0·521	0·746	0·678	0·728	− 9·1	−2·4
1P24	0·25	16·91	0·553	0·604	0·673	0·601	+11·4	−0·5
2P24	0·46	20·75	0·489	0·635	0·699	0·639	+10·1	+0·6
3P24	0·78	27·87	0·636	0·694	—	0·702	—	+1·2
4P24	1·43	40·43	0·535	0·758	0·830	0·778	+ 9·5	+2·6
1D24	−1·80	10·29	0·518	0·661	0·632	0·652	− 4·4	−1·4
2D24	−1·55	20·14	0·531	0·761	0·753	0·786	− 1·1	+3·3
3D24	−1·85	10·28	0·458	0·657	0·624	0·647	− 5·0	−1·5
4D24	−1·59	20·17	0·438	0·760	0·735	0·779	− 3·3	+2·5
1PN47	0·57	35·04	1·000	0·627	—	0·618	—	−1·4
3PN47	0·51	35·13	1·000	0·604	—	0·600	—	−0·7
4PN47	0·62	43·02	1·000	0·651	—	0·647	—	−0·6
5PN47	0·95	52·09	1·000	0·694	—	0·700	—	+0·9
6PN47	1·27	59·27	1·000	0·740	—	0·736	—	−0·5
4PN47	1·24	68·52	1·000	0·770	—	0·778	—	+1·0
8PN47	2·00	74·43	1·000	0·806	—	0·807	—	+0·1
9PN47	2·52	84·94	1·000	0·836	—	0·850	—	+1·7

All experiments were evaluated by comparing the experimental porosity of the moving fluidized bed $\varepsilon_{M(E)}$ with the theoretical (calculated) value $\varepsilon_{M(C)}$. The theoretical value $\varepsilon_{M(C)}$ for experimental conditions Y_J, u_c, w_f was calculated from Eq. (*2*). written for both components of the binary mixture together with the appropriate expansion function, Eq. (*1*), obtained earlier, and with the aid of Eq. (*3*). In addition, the value of $\varepsilon_{M(C)}$ was calculated by the approach of the authors[3], *i.e.* with the aid of Eqs (*2a*), (*4*)–(*6*). The latter processing was applied only to a part of the experiments. The PN type experiments were not processed by this latter method since for this case it is clearly incorrect. According to this approach[3] a monodisperse feed can give rise only to an moving fluidized bed of identical composition. However, with the presence of another non-moving advancing component the composition of the solids is binary. Of the remaining 20 experiments 14 were evaluated according to the authors[3]. For simplicity we processed only those experiments in which the expansion of fluidized bed of both the fine and coarse component occurred in the same hydrodynamic region[9].

The relative deviations Δ_M of the theoretical values $\varepsilon_{M(C)}$ from the experimental ones $\varepsilon_{M(E)}$ were computed from

$$\Delta_M = (\varepsilon_{M(C)} - \varepsilon_{M(E)})/\varepsilon_{M(E)}\ 100\,. \tag{7}$$

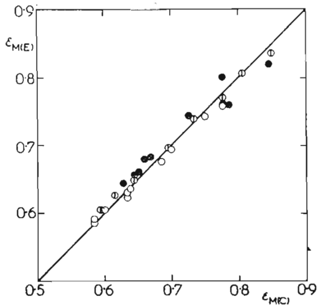

FIG. 2

A plot of Experimental Porosity $\varepsilon_{M(E)}$ *versus* Theoretical Value $\varepsilon_{M(C)}$ Computed from Eq. (2) for Binary Mixtures of Sand

○ Co-current, ● counter-current arrangement, ⊖ moving bed of finer component in co-current arrangement with nonmoving ing bed of coarse component.

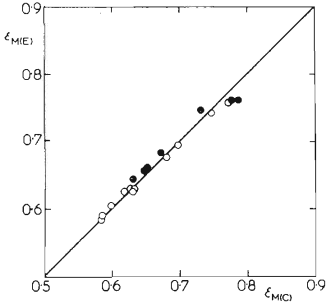

FIG. 3

A Plot of $\varepsilon_{M(E)}$ *versus* $\varepsilon_{M(C)}$ for Binary Mixtures of Sand with Correction on the Change of d_e of Individual Fractions in the Fluidized Bed with Respect to the Feed

○ Co-current, ● counter-current arrangement.

The results calculated according to the authors[3] and Eq. (*2*) are both summarized in Table I together with the experimental ones.

DISCUSSION

By comparing the experimental values of the mass fraction of the finer component in the feed, Y_J, and the solid phase of the fluidized bed, X_J,it was found in accord with Eq.(*2*), that: for co-current arrangement $Y_J > X_J$ and for counter-current arrangement $Y_J < X_J$. This finding is fully consistent with the function, Eq. (*2b*), plotted in Fig. 1.

From the results shown in Table I it further follows that the theoretical values according to the authors[3] display in average higher value of Δ_M (the estimated standard deviation of Δ_M amounts to 8·7%) than those obtained with the aid of Eq. (*2*) (for the same14 experiments it is 2%). From the results in Table I it can also be seen that the approach of the authors[3] carries a systematic error which is positive under co-current arrangement and negative under counter-current arrangement of the flow. In contrast, the error of the calculations based on Eq. (*2*) is random.

The authors[3] assumed no segregation of the mixture after it had entered the moving fluidized bed. In our experiments it was found that starting already from the distance 100 mm from the feed of the solid binary mixture the porosity of the fluidized bed remains constant.

The average residence time of the particles, $\bar{\tau}$, in the bed of height h may be estimated from

$$\bar{\tau} = (1 - \varepsilon)\, h/u_c\,. \tag{8}$$

For the experiments tabulated in Table I and the height 100 mm the value of $\bar{\tau}$ amounts to 10–20 s. The particle segregation thus must have taken place within this period. This again strengthens the argument against the estimated 60 seconds within which, according to authors[3], no segregation takes place. According to the work[3], no segregation will take place within 60 seconds if the coarse to fine particles diameter ratio is smaller than 3. Even this restriction though is not sufficient since the pertaining ratio in our experiments was 1·6 (the binary F2 + F4) or 1·9 (the binary F3 + F6).

A comparison of the experimental porosity $\varepsilon_{M(E)}$ with the results $\varepsilon_{M(C)}$ computed using Eq. (*2*) is shown in Fig. 2. The porosities $\varepsilon_{M(C)}$ plotted in Fig. 3 were computed with a correction on the change of the mixture equivalent diameter in a fluidized bed with respect of the fed solids.* The purpose of Fig. 3 is to illustrate the effect of

* Since the binary mixtures were not composed of two monodisperse materials but rather narrow fractions, the properties of individual fractions may change in time due to the different properties of individual particles forming the fluidized bed. This phenomenon was observed during experiments by sampling the fluidized bed[1].

this correction. The figure indicates that the corrected values of the porosity of the moving fluidized bed approximate even better the experimentally found porosities. The change, however, is not very conspiscuous as the fractions of the sand used were nearly monodisperse particle sets.

It may be concluded that our experiments did not confirm the approach of the authors[3]. The segregation is apparently completed in moving fluidized bed much closer to the feed of the granular particles. Even though the kinetics of segregation was not studied, our experiment suggest that the segregation was completed within 10–20 seconds after the material had entered the fluidized bed for the coarse to fine particle diameter ratios smaller than 3. In all experiments with the moving fluidized bed of binary mixtures of granulated material it turned out that the deviation of Eq. (*2*), Δ_M, does not exceed 3·3%. Eq. (*2*) can thus be used for the calculation of expansion of an moving fluidized bed both for the monodisperse and polydisperse materials from the knowledge of expansion of non-moving fluidized bed.

With the aid of the set of equations expressed by Eq. (*2*) and the pertaining expansion functions (*1*) one can also obtain information about the composition of the granulated material in the moving fluidized bed from the experimental data on its porosity.

LIST OF SYMBOLS

$\mathrm{Ar} = g d_e^3(\varrho_J - \varrho_f)/\nu^2 \varrho_f$ Archimedes number
d_a average particle diameter in the fluidized bed
d_e equivalent particle diameter by weighing
g acceleration due to gravity
h height of fluidized bed
m mass
N number of particles
$\mathrm{Re} = d_e w/e$ Reynolds number
S column cross section area
u_c throughput of granulated material defined as linear velocity of compact layer ($\varepsilon = 0$)
$\dot{V}$ volume flow rate
w average superficial velocity of fluid with respect to particles in ideally uniform moving fluidized bed
w_f average relative superficial velocity of fluid
X_i volume fraction of of *i*-th component polydisperse material (solid) in fluidized bed
Y_i volume fraction of *i*-th component of fed polydisperse material
ε porosity of fluidized bed
ϱ density
ν kinematic viscosity
Δ_M deviation defined by Eq. (*7*)
$\bar{\tau}$ residence time

Subscripts

C computed
E experimental
f fluid
H coarse component of binary mixture
i sequence index of component
J fine component of binary mixture
M moving fluidized bed
s solid phase

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